

# Angular measurement error budget and uncertainty calculations

## Introduction

This document summarises the system measurement uncertainty calculations for the Renishaw standard (0.2%) and calibrated interferometric angular optics when used with a Renishaw XL-80 or ML10 laser. These calculations are used to derive the published specifications:

- The uncertainty of measurement of *standard angular optics* is:  $\pm 0.2\% \theta \pm 0.5 \pm 0.1M \mu\text{rad}$ .
- The uncertainty of measurement of *calibrated angular optics* is:  $\pm 0.02\% \theta \pm 0.5 \pm 0.1M \mu\text{rad}$ .

Where:  $\theta$  is the angle measured in  $\mu\text{rad}$ .

M is the linear measurement distance between the angular interferometer and angular reflector in metres.

The first terms (i.e.  $0.2\% \theta \mu\text{rad}$  and  $0.02\% \theta \mu\text{rad}$ ) in the specifications above are known as the 'angular factors' expressed as percentages and depend on the measured centre to centre separation, L of the retroreflectors divided by the theoretical retroreflector separation of 30 mm.

The second term (i.e.  $0.5 \mu\text{rad}$ ) in the specifications above is known as the interpolation error term and results from the performance of the interferometer optics.

The third term (i.e.  $0.1M \mu\text{rad}$ ) in the specifications above is known as the cosine error term and is due to the non-parallelism of the laser beams.

**Note:** the above overall uncertainties do not include the temperature sensitivity contribution of the angular interferometer. This contribution can be minimised by a tight control of the environmental temperature stability. See White paper: [Interferometric angle measurement and the Hardware options available from Renishaw](#) (TE 326) for further information.

## General information on the methodology used

The derivation of error budgets is a standard part of Renishaw's new product development process which is covered by the group's ISO 9001:2008 compliant quality management system. ISO 9001:2008 is the internationally recognised standard for quality management systems and is verified at Renishaw by BSI management systems (UKAS accredited).

Error budgets are calculated following the guidance as outlined in EA-4/02 'Expression of the uncertainty of measurement in calibration' and NIST technical note 1297. The contributory elements to the budget are derived from a combination of the validation of component specifications, experimental evidence of performance and theoretical calculations. The combination of these individual terms results in the system level specification. All specifications are published with 95% ( $k=2$ ) confidence level.

The error budgets are reviewed and signed off by qualified personnel. The sections on page 2 show the individual elements of these error budgets.

## Uncertainty in the measurement of angles with standard angular optics

Source of uncertainty	Uncertainty value ( $\pm$ )	Probability distribution	Uncertainty (k=1) ( $\pm\mu\text{rad}$ )
Retroreflector separation (L)	0.2% L mm	Normal (k=2)	0.1% $\theta$
XL temperature range	0 - 40°C	Normal (k=2)	0.029% $\theta$
Repeatability of measurement	0.006% $\theta$ $\mu\text{rad}$	Normal (k=2)	Negligible
Laser frequency	0.05ppm	Normal (k=2)	Negligible
Interpolation error	0.50 $\mu\text{rad}$	Normal (k=2)	0.25
Cosine error	0.10M $\mu\text{rad}$	Normal (k=2)	0.05M
Combined uncertainty (k=1)			$\pm 0.1\%\theta \pm 0.25 \pm 0.05M \mu\text{rad}$
Expanded uncertainty (k=2)			$\pm 0.2\%\theta \pm 0.5 \pm 0.1M \mu\text{rad}$
Published specification (k=2)			$\pm 0.2\%\theta \pm 0.5 \pm 0.1M \mu\text{rad}$

If an angle of 100  $\mu\text{rad}$  is measured with the angular optics separated by a distance of 0.5 m, the uncertainty of measurement is  $\pm 0.8 \mu\text{rad}$ .

If an angle of 1000  $\mu\text{rad}$  is measured with the angular optics separated by a distance of 1 m, the uncertainty of measurement is  $\pm 2.6 \mu\text{rad}$ .

## Uncertainty in the measurement of angles with calibrated angular optics

Individually calibrated systems are supplied with a calibration certificate stating the angular factor.

This value is entered into the Renishaw angular measurement software to improve the accuracy of the measurement.

Source of uncertainty	Uncertainty value ( $\pm$ )	Probability distribution	Uncertainty (k=1) ( $\pm\mu\text{rad}$ )
Uncertainty of the calibration*	0.02% L mm	Normal (k=2)	0.01% $\theta$
Repeatability of measurement	0.006%A $\mu\text{rad}$	Normal (k=2)	Negligible
Laser frequency	0.05ppm	Normal (k=2)	Negligible
Interpolation error	0.50 $\mu\text{rad}$	Normal (k=2)	0.25
Cosine error	0.10M $\mu\text{rad}$	Normal (k=2)	0.05M
Combined uncertainty (k=1)			$\pm 0.01\%\theta \pm 0.25 \pm 0.05M \mu\text{rad}$
Expanded uncertainty (k=2)			$\pm 0.02\%\theta \pm 0.5 \pm 0.1M \mu\text{rad}$
Published specification (k=2)			$\pm 0.02\%\theta \pm 0.5 \pm 0.1M \mu\text{rad}$

\*Valid for temperature range of 15-25°C

If an angle of 100  $\mu\text{rad}$  is measured with the angular optics separated by a distance of 0.5 m, the uncertainty of measurement is  $\pm 0.6 \mu\text{rad}$ .

If an angle of 1000  $\mu\text{rad}$  is measured with the angular optics separated by a distance of 1 m, the uncertainty of measurement is  $\pm 0.8 \mu\text{rad}$ .

**Note:** Specified measurement uncertainties do not make allowance for errors in the alignment and set up of measuring optics or in the implementation of the measuring procedure, both of which can affect the accuracy of a practical measurement. See system manuals for further details.

### General notes for all tables:

- The first column lists the sources of the uncertainty errors considered.
- The second column gives the uncertainty value for the sources of uncertainty in the appropriate units.
- The third column gives the probability distribution function for the source of the errors and the coverage factor used. The coverage factor, k is used to convert uncertainty values to k=1. An additional multiplier is applied to account for size of the sample data where necessary.
- The fourth column lists the sources of uncertainty. The uncertainties of the angular factor terms are listed, followed by the offset terms. The angular factor terms are combined using a 'root sum square' calculation to produce the total uncertainty for the angular factor. This is combined with the offset terms to give the total uncertainty of the measured angle.

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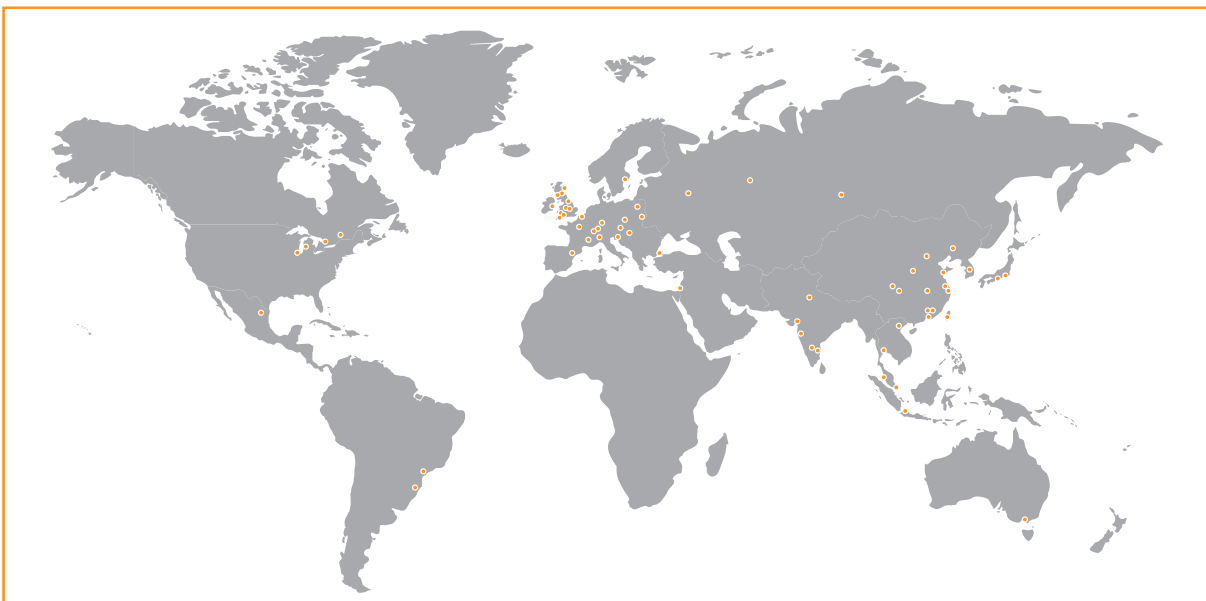
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